

Calculators and mobile phones are not allowed.

Answer all of the following questions.

1. Evaluate the following integrals

(a) $\int e^{-2x} \sin x \, dx,$

(b) $\int \frac{\cos^3 x}{\sqrt{\sin x}} \, dx,$

(c) $\int \frac{dx}{\sqrt{2x - x^2}},$

(d) $\int \frac{dx}{\sin x - \cos x + 1}$

7 points each

2. Determine whether the improper integral

$$\int_0^1 \ln(x^x) \, dx$$

is convergent or divergent and if convergent find its value.

7 points

3. Find the arc length of the parametric curve

$$x = t + 2, \quad y = \cosh t, \quad \ln 2 \leq t \leq \ln 3.$$

7 points

4. Sketch the polar curves $r = 3 \cos \theta$ and $r = 2 - \cos \theta$ and find the area that is inside the graph of $r = 3 \cos \theta$ and outside the graph of $r = 2 - \cos \theta$.

8 points